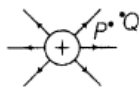


Chapter 2. Electrostatic Potential and Capacitance

Electrostatic Potential

1 Mark Questions

1. The figure shows the field lines of a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? [Foreign 2014]



Ans.

Work done by charge is given by

$$W = q (\text{potential at Q} - \text{potential at P}).$$

where, q = small positive charge (1/2)

The electric potential at a point distant r due to the field created by a positive charge Q is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$\because r_p < r_Q$$

$$\Rightarrow V_p > V_Q \quad (1/2)$$

So, work done will be negative.

2. For any charge configuration, equipotential surface through a point is a normal to the electric field. Justify. [Delhi 2014]

Ans.

No work is done in moving the test charge from one point of an equipotential surface to the other. (1/2)

$$\therefore W_B - W_A = 0 = -\int \mathbf{E} \cdot d\mathbf{l}$$

$$\Rightarrow \mathbf{E} \cdot d\mathbf{l} = 0$$

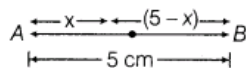
$$\text{Hence, } \mathbf{E} \perp d\mathbf{l} \quad (1/2)$$

3. Two charges $2 \mu\text{C}$ and $-2 \mu\text{C}$ are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system. [Delhi 2013C]

Ans.

$$\text{Given, } q_A = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$$

$$q_B = -2 \mu\text{C} = -2 \times 10^{-6} \text{ C}$$



$$\text{and } r = 5 \text{ cm}$$

$$\begin{aligned} \therefore \text{Potential, } V &= \frac{2 \times 10^{-6}}{4\pi\epsilon_0 x \times 10^{-2}} \\ &\quad + \frac{-2 \times 10^{-6}}{4\pi\epsilon_0 (5-x) \times 10^{-2}} \\ \therefore \frac{2 \times 10^{-6}}{4\pi\epsilon_0 x \times 10^{-2}} &= \frac{2 \times 10^{-6}}{4\pi\epsilon_0 (5-x) \times 10^{-2}} \quad [\because V = 0] \\ x &= 5 - x \\ x &= 2.5 \quad (1) \end{aligned}$$

4. Why electrostatic potential is constant throughout the volume of the conductor and has the same value as on its surface? [Delhi 2012]

Ans. Since, electric field intensity inside the conductor is zero. So, electrostatic potential is a constant.

$$\begin{aligned} \text{But,} \quad E &= -\frac{\Delta V}{\Delta r} \\ \therefore \quad E &= 0, \quad \Delta V = 0 \\ \text{or} \quad V_2 - V_1 &= 0, \quad V_2 = V_1 \end{aligned}$$

5. Why is the potential inside a hollow spherical charged conductor is constant and has the same value as on its surface? [Foreign 2012]


Ans. Electric field inside the hollow spherical charged conductor is zero. So, no work is done in moving a charge inside the shell.

This implies that potential is a constant and therefore, equal to its value at the surface, i.e.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

6. Why there is no work done in moving a charge from one point to another on an equipotential surface? [Foreign 2012]

Ans.

 An equipotential surface is a surface at which every point electric potential is same.

As, work done in moving a charged particle from one point to another is defined as


$$\Delta W = q(\Delta V)$$

On an equipotential surface, the potential remains constant. So, $\Delta V = \text{zero}$

$$\Rightarrow \text{Work done, } \Delta W = 0 \quad (1)$$

7. A hollow metal sphere of radius 5 cm is charged such that potential on its surface is 10 V. What is the potential at the centre of the sphere? [All India 2011]

Ans.

 The electric potential at every point inside the charged spherical shell is same and equal to the electric potential on its surface.

The electric potential at the centre of sphere is 10 V. (1)

8. Can two equipotential surface intersect each other? Justify your answer. [Delhi 2011 c]

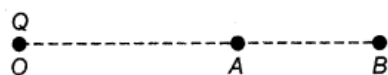
Ans. (i) No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on two surfaces which give two directions of E at the same point which is impossible.

(ii) Also, two values of potential at the same point is not possible.

9. A point charge Q is placed at point O as shown in the figure. Is the potential difference

$$(V_A - V_B)$$

positive, negative or zero if Q is (i) positive (ii) negative



[All India 2011]

Ans.

Let the distance of points A and B from charge Q be r_A and r_B , respectively.

∴ Potential difference between points A and B

$$V_A - V_B = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

As, $r_A = OA$, $r_B = OB$ and $r_A < r_B$, $\frac{1}{r_A} > \frac{1}{r_B}$

Therefore, $\left[\frac{1}{r_A} - \frac{1}{r_B} \right]$ has positive value.

($V_A - V_B$) depends on the nature of charge q .

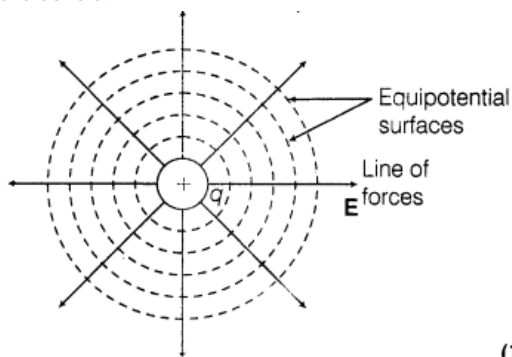
(i) ($V_A - V_B$) is positive when $Q > 0$

(ii) ($V_A - V_B$) is negative when $Q < 0$

$$(1/2 + 1/2 = 1)$$

10. Draw equipotential surfaces due to a single point charge. [All India 2011 c]

Ans. Equipotential surfaces due to a single point charge are concentric sphere having charge at the centre.



(1)

11. Name the physical quantity whose SI unit is J/C. Is it a scalar or a vector quantity? [All India 2010]

Ans. J/C is the SI unit of **electric** potential. It is a scalar quantity.

12. What is the work done in moving a test charge q through a distance of 1 cm along the equatorial axis of an electric dipole? [All India 2009]

Ans.

💡 The electric potential at any point on equatorial line due to an electric dipole is equal to zero.

No work is done in moving a test charge along equatorial line, as given below:

$$W = q\Delta V = q(V_B - V_A)$$

$$\therefore V_B = V_A = 0$$

$$W = 0$$

(1)

13. Define the term potential energy for charge q at a distance r in an external field. [All India 2009]

Ans. The electric potential energy at any point lying at a distance r from the source charge q is equal to the amount of work done in moving unit positive test charge from infinity to that point without any acceleration against of electrostatic force.

14. The potential due to a dipole at any point on its axial line is zero. [All India 2009 C]

Ans. Wrong, as potential due to an electric dipole is zero on equatorial line in spite of axial line. The potential due to a dipole at any point on equatorial line is zero, not an axial line.

15. What is the electric potential due to an electric dipole at an equatorial point? [All India 2009]

Ans. Zero, as potential on equatorial point due to charges of electric dipole, is equal to magnitude but opposite in nature and hence, their resultant is zero

16. A $500\text{ }\mu\text{C}$ charge is at the centre of a square of side 10 cm . Find the work done in moving a charge of $10\text{ }\mu\text{C}$ between two diagonally opposite points on the square. [Delhi 2008]

Ans.

Two diagonally opposite points on the square are equidistant from the centre of the square. Therefore, potential at these points due to charge q will be equal to each other.

Work done, $W = q\Delta V$

$$W = q \times 0$$

$$W = 0$$

[$\Delta V = 0$ as the two diagonally opposite points are at the same potential due to $500\text{ }\mu\text{C}$ charge]. (1)

2 Marks Questions

17. Two point charges q_1 and q_2 are located at r_1 and r_2 , respectively in an external electric field E . Obtain the expression for the total work done in assembling this configuration. [Delhi 2014 C]

Ans.

Work done in bringing the charge q_1 from infinity to position r_1

$$W_1 = q_1 V(r_1) \quad \dots (i) \quad (1/2)$$

Work done in bringing charge q_2 to the position r_2

$$W_2 = q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad \dots (ii) \quad (1/2)$$

Hence, total work done in assembling the two charges.

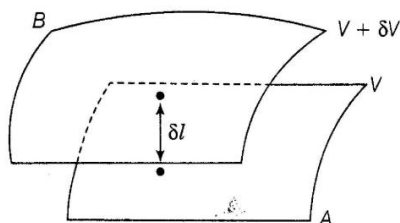
$$W = W_1 + W_2 \quad (1)$$

From Eqs. (i) and (ii), we get

$$W = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$



18. Two closely spaced equipotential surfaces A and B with potentials V and $V + \delta V$, (where δV is the change in V) are kept δl distance apart as shown in the figure. Deduce the relation between the electric field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potential. [Delhi 2014C]



Ans.

Work done in moving a unit positive charge along distance δl ,

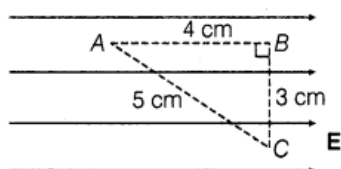
$$|E_l| \delta l = V_A - V_B = V - (V + \delta V) = -\delta V$$

$$E = \frac{-\delta V}{\delta l} \quad (1)$$

19. A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure.

(i) Calculate the potential difference between A and C.

(ii) At which point (of the two) is the electric potential more and why?



[Foreign 2009; All India 2012]

Ans.

- (i) \therefore Electric field intensity and potential difference are related as

$$E = -\frac{\Delta V}{\Delta r}$$

$$\Delta V = -E \Delta r$$

$$\Rightarrow V_C - V_A = -4E \quad (1)$$

$$\left[\begin{array}{l} \therefore \text{By pythagoras law, } AC^2 = AB^2 + BC^2 \\ \Rightarrow AB^2 = 5^2 - 3^2 \\ \Delta r = \sqrt{16} \\ \Delta r = 4 \end{array} \right]$$

- (ii) As $V_C - V_A = 4E$ is negative (1)

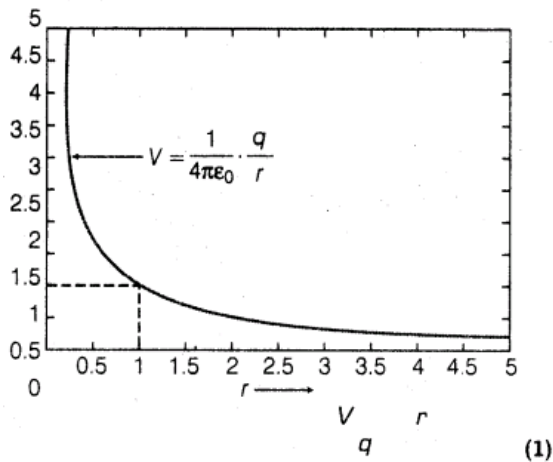
$$\therefore V_C < V_A$$

Potential is greater at point A than point C, as potential decreases along the direction of electric field.

20. Graph the electric potential (V) with distance r due to a point charge Q . [Delhi 2012]

Ans.

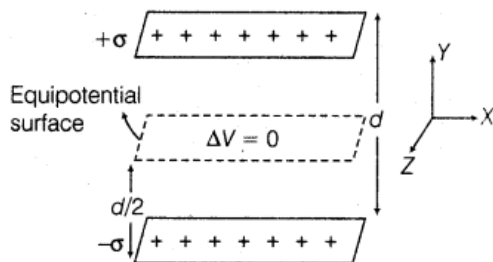
The graph is shown as below:



Potential will be maximum where r will be minimum (not zero). Zero potential assumed to be at infinity, i.e. at large distance. (1)

- 21.** Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the XZ - plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates. What is the magnitude and direction of this field? [Delhi 2011]

Ans.



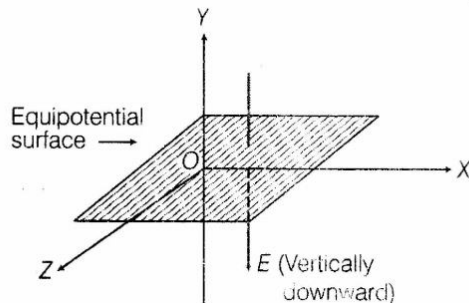
The equipotential surface is at a distance $d/2$ from either plate in XZ -plane. $-q$ charge experiences a force in a direction opposite to the direction of electric field. (1)

$\therefore -q$ charge balances when

$$qE = mg$$

$$E = \frac{mg}{q}$$

The direction of electric field along vertically downward direction. The XZ -plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight (mg) of charge particle could not be balanced. (1)



22. A dipole with its charge $-q$ and $+q$ located at the points $(0, -b, 0)$ and $(0, +b, 0)$ is present in a uniform electric field E . The equipotential surfaces of this field are planes parallel to the YZ -plane.

(i) What is the direction of the electric field E ?

(ii) How much torque would the dipole experience in this field? [Delhi 2010 C]

Ans.

The direction of electric field is perpendicular to the equipotential surface.

(i) The direction of electric field is along X -axis as it should be perpendicular to equipotential surface lying in YZ -plane.

Length of the dipole $= 2b$

As dipole's axis is along the Y -axis.

\therefore Electric dipole moment

$$\mathbf{p} = q(2b)\hat{\mathbf{j}} \quad (1)$$

(ii) Electric field, $\mathbf{E} = E\hat{\mathbf{i}}$

$$\begin{aligned} \therefore \tau &= \mathbf{p} \times \mathbf{E} \\ &= q(2b)\hat{\mathbf{j}} \times E\hat{\mathbf{i}} \\ &= +2qbE(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) \\ &= 2qbE(-\hat{\mathbf{k}}) \end{aligned}$$

$$\therefore \text{Torque } |\tau| = 2qbE \quad (1)$$

Alternative method

\mathbf{E} is directed along X -axis.

Dipole moment $p = q(2b)$ from $(0, -b, 0)$ to $(0, b, 0)$, i.e. along Y -axis. (1)

\therefore Angle between \mathbf{p} and \mathbf{E} is 90°

$$\begin{aligned} \therefore \text{Torque on dipole} &= \tau_{\max} = pE \sin 90^\circ \\ &= q(2b)E \times 1 \end{aligned}$$

$$\therefore \text{Torque } \tau = 2qbE \quad (1)$$

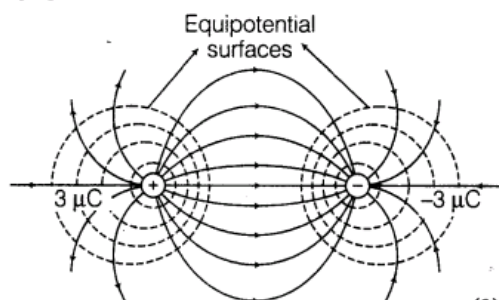
23. Two point charges $3\ \mu\text{C}$ and $-3\ \mu\text{C}$ are placed at points A and B, 5 cm apart.

(i) Draw the equipotential surfaces of the system.

(ii) Why do equipotential surfaces get close to each other near the point charge? [All India 2011C]

Ans.

(i) Equipotential surfaces



(1)

(ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

$$E = -\frac{\Delta V}{\Delta r} \text{ and } E \propto -\frac{1}{\Delta r}$$

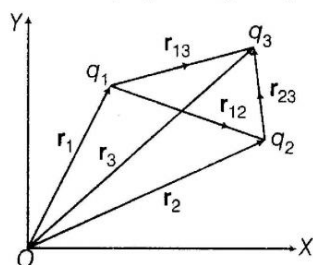
For a given equipotential surface, small Δr represents strong electric field and vice-versa.

(1)

24. Find out the expression for the potential energy of a system of three charges q_1 , q_2 and q_3 located at \mathbf{r}_1 , \mathbf{r}_2 and \mathbf{r}_3 with respect to the common origin O. [Delhi 2010 c]

Ans.

Let three point charges q_1 , q_2 and q_3 have position vectors \mathbf{r}_1 , \mathbf{r}_2 and \mathbf{r}_3 , respectively.



Potential energy of the charges q_1 and q_2 ,

$$U_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_{12}|} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

Similarly,
$$U_{23} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|}$$

$$U_{31} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|} \quad (1)$$

\therefore Net potential energy of the system,

$$U = U_{12} + U_{23} + U_{31}$$

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|} + \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|} + \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|} \right] \quad (1)$$

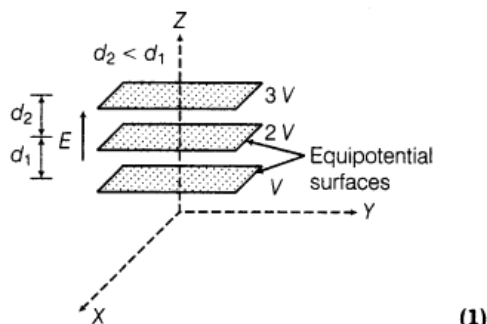
25. Draw three equipotential surfaces corresponding to a field that uniformly increases in

magnitude but remains constant along Z-direction. How are these surfaces different from that of a constant electric field along Z-direction? [Foreign 2008; All India 2009]

Ans.

💡 Electric field is always directed from higher to lower potential and perpendicular to the equipotential surfaces.

The figure is shown as below :



In case of constant electric field along Z-direction, the perpendicular distance between equipotential surfaces remains same whereas for field of increasing magnitude, equipotential surfaces get closer as we go away from the origin. In both cases, surfaces be planes parallel to XY- plane.

26.(i) Can two equipotential surfaces intersect each other? Give reasons,

(ii) Two charges $-q$ and $+q$ are located at points A $(0, 0, -a)$ and B $(0, 0, +a)$ respectively.

How much work is done in moving a test charge from point P $(7, 0, 0)$ to Q $(-3, 0, 0)$? [Delhi 2009]

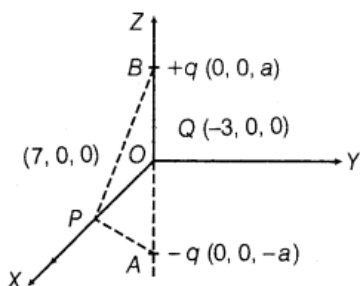
Ans. (i) No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on two surfaces which give two directions of E at the same point which is impossible.

(ii) \therefore Every point on X-axis is on equatorial line of electric dipole (system of two unlike charges).

\therefore Therefore, potential on it is equal to zero.

\therefore No work is done in moving a test charge.

$$W = q \Delta V = q \times 0 = 0. \quad [\because \Delta V = 0]$$



\therefore Work done in moving a charge on an equipotential surface is zero. (1)

27.(i) Can two equipotential surfaces intersect each other? Give reasons, (ii) Two charges $+q$ and $-q$ are located at points A $(0, 0, -2)$ and B $(0, 0, 2)$ respectively. How much work will be done in moving a test charge from point P $(7, 0, 0)$ to Q $(-3, 0, 0)$? [Delhi 2009]

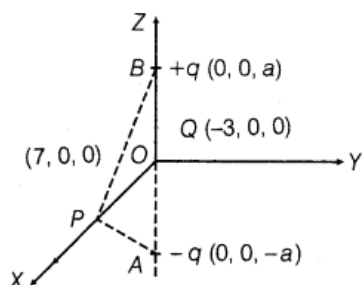
Ans. (i) No, two equipotential surfaces cannot intersect each other because two normals can be drawn at intersecting point on two surfaces which give two directions of E at the same point which is impossible.

(ii) \therefore Every point on X-axis is on equatorial line of electric dipole (system of two unlike charges).

\therefore Therefore, potential on it is equal to zero.

\therefore No work is done in moving a test charge.

$$W = q \Delta V = q \times 0 = 0. \quad [\because \Delta V = 0]$$



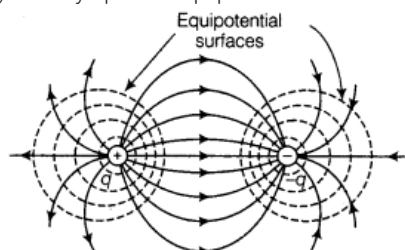
\therefore Work done in moving a charge on an equipotential surface is zero. (1)

28.(i) Write two characteristics of equipotential surfaces.

(ii) Draw the equipotential surfaces due to an electric dipole. [All India 2009 C]

Ans.(i) Two equipotential surfaces do not intersect each other as normals at intersecting points on two surfaces will give two directions of electric field which is impossible.

(ii) Closely spaced equipotential surfaces represent strong electric field and vice-versa.



Equipotential surfaces due to an electric dipole

29. Two point charges, $q_1 = 10 \times 10^{-8} \text{ C}$ and $q_2 = -2 \times 10^{-8} \text{ C}$ are separated by a distance of 60 cm in air.

(i) What a distance from the 1st charge q_1 would the electric potential be zero?

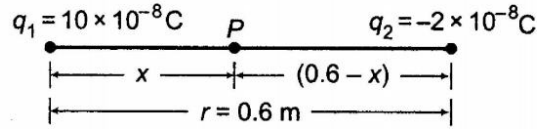
(ii) Also, calculate the electrostatic potential energy of the system. [All India 2007, 2008]

Ans.



The electric potential is a scalar quantity. The total potential at a point due to different charges is equal to the sum of individual potentials due to them.

- (i) Let electric potential due to two point charges q_1 and q_2 is zero at point P which lie at distance x from q_1 .



$$\begin{aligned}
 V_1 + V_2 &= 0 \\
 \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{(0.6 - x)} &= 0 \quad (1/2) \\
 \Rightarrow \frac{q_1}{x} + \frac{q_2}{(0.6 - x)} &= 0 \\
 \Rightarrow \frac{10 \times 10^{-8}}{x} + \frac{(-2 \times 10^{-8})}{(0.6 - x)} &= 0 \\
 \Rightarrow \frac{10}{x} = \frac{2}{(0.6 - x)} \\
 6 - 10x &= 2x \Rightarrow 12x = 6 \\
 x &= \frac{1}{2} \text{ m} \\
 x &= 50 \text{ cm} \quad (1/2)
 \end{aligned}$$

Therefore, electric potential is zero at a distance 50 cm from charge q_1 between two charges.

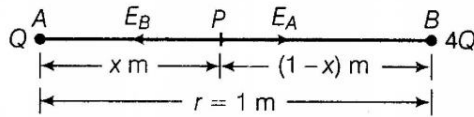
$$\begin{aligned}
 (ii) \text{ Potential energy, } U &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} \\
 &= 9 \times 10^9 \times \frac{(10 \times 10^{-8})(-2 \times 10^{-8})}{0.6}
 \end{aligned}$$

$$\begin{aligned}
 U &= -3 \times 10^{-5} \text{ J} \\
 &= -30 \mu\text{J}
 \end{aligned}$$

30. Two point charges 40, 0 are separated by 1m in air. At what point on the line joining the charges, is the electric field intensity zero? Also calculate the electrostatic potential energy of the system of charges taking the value of charge, 0 = $2 \times 10^{-7} \text{ C}$. [All India 2008]

Ans.

Let electric field intensity be zero at any point P lying at a distance x m from charge Q between the two charges on the line joining them.



Therefore, at point P

$$E_A = E_B$$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{x^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Q}{(1-x)^2}$$

$$\Rightarrow \frac{1}{x^2} = \frac{4}{(1-x)^2}$$

$$\left(\frac{1-x}{x}\right)^2 = 4 \quad \text{or} \quad \frac{1-x}{x} = 2$$

$$3x = 1$$

$$x = \frac{1}{3} \text{ m} \quad (1)$$

Electric field intensity E is zero at a point lying $x = \frac{1}{3}$ m from $+Q$ charge on the line joining two charges.

Now, electrostatic potential energy

$$U = \frac{1}{4\pi\epsilon_0} \frac{(4Q)(Q)}{r}$$

$$U = 9 \times 10^9 \times \frac{4 \times (2 \times 10^{-7})^2}{1}$$

$$U = 1.44 \times 10^{-3} \text{ J} = 1.44 \text{ mJ} \quad (1)$$

31. Define the dipole moment of an electric dipole. How does the electric potential due to a dipole vary on the dipole axis as a function of r distance of the point from the mid-point of the dipole at large distances? [All India 2008 C]

Ans. Electric field inside the hollow spherical charged conductor is zero. So, no work is done in moving a charge inside the shell.

This implies that potential is a constant and therefore, equal to its value at the surface, i.e.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

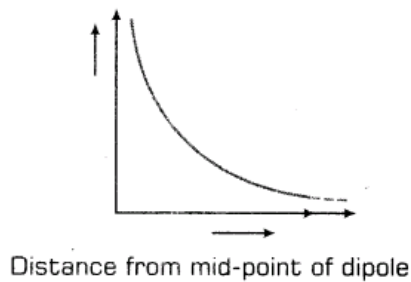
variation of electric potential due to dipole on axial lines when point is at large distance from mid-point of the dipole, occurs as per relation below.

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$$

For given dipole, $\frac{p}{4\pi\epsilon_0} = \text{constant}$

$$\Rightarrow V \propto \frac{1}{r^2} \quad (1)$$

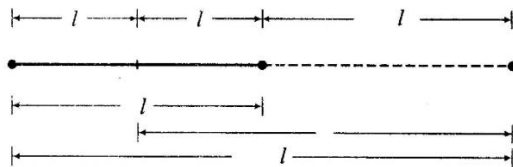
The variation of electric potential with distance r is shown as below:



32. Derive an expression for the electric potential at any point along the axial line of an electric dipole. [Delhi 2008]

Ans.

Consider an electric dipole AB consists of two point charges $+q$ and $-q$ separated by a distance $2l$. A point P be on its axis separated by a distance r from dipole's centre O .



Electric dipole moment,

$$p = q(2l) \quad \dots(i)$$

[from $-q$ charge to $+q$ charge]

Now, electric potential V_A at point P due to $+q$ charge

$$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-l)} \quad \dots(ii)$$

Similarly, electric potential V_B at point P due to $-q$ charge

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{-q}{(r+l)} \quad \dots(iii)$$

\therefore Net electric potential at point P due to two charges

$$\begin{aligned} V &= V_A + V_B = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-l)} - \frac{1}{(r+l)} \right] \\ V &= \frac{q}{4\pi\epsilon_0} \left[\frac{(r+l) - (r-l)}{(r-l)(r+l)} \right] \\ V &= \frac{q}{4\pi\epsilon_0} \frac{(2l)}{(r^2 - l^2)} \quad \dots(iv) \end{aligned}$$

[from Eq. (i) $p = q(2l)$]

When $l \ll r$,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \quad \dots(v)$$

(1)

33. Two charges of 5 nC and -2 nC are placed at points $(5 \text{ cm}, 0, 0)$ and $(23 \text{ cm}, 0, 0)$ in the region of space, where there is no other external field. Calculate the electrostatic potential energy of this charge system. [Delhi 2008 C]

Ans.

Given, $q_1 = 5 \times 10^{-9} \text{ C}$
 $r_1 = 5 \times 10^{-2} \hat{i} \text{ m}$
 $q_2 = -2 \times 10^{-9} \text{ C}$
and $r_2 = 23 \times 10^{-2} \hat{i} \text{ m}$
 $\therefore r_2 - r_1 = (23 - 5) \times 10^{-2} \hat{i}$
 $= 0.18 \hat{i} \text{ m}$
 $\Rightarrow |r_2 - r_1| = 0.18 \text{ m} \quad (1/2)$
 \therefore Electrostatic potential energy, $(1/2)$

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r}$$

$$= 9 \times 10^9 \times \frac{(5 \times 10^{-9})(-2 \times 10^{-9})}{0.18}$$

$$U = -5.0 \times 10^{-7} \text{ J}$$

$$U = -0.5 \mu\text{J} \quad (1)$$

3 Marks Questions

34. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5 A is held directly above AB at a height of 1 mm. Find the mass per unit length of the wire CD, so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in [Take the value of $g = 10 \text{ ms}^{-2}$] [All India 2013]

Ans.

Force per unit length between the current carrying wires is given as

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$

where, I_1 = current in wire, AB = 12 A and
 I_2 = current in wire, CD = 5 A and r = distance
between wires = 1 mm = $1 \times 10^{-3} \text{ m}$

$$\therefore \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r} = mg$$

Here, m is mass per unit length.

$$\Rightarrow 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} = m \times 10$$

$$\Rightarrow m = 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} \times \frac{1}{10}$$

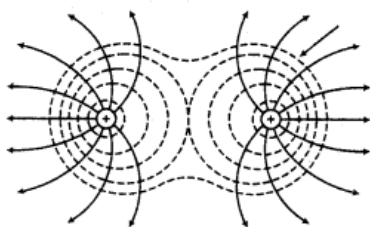
$$= 1.2 \times 10^{-3} \text{ kg/m} \quad (3)$$

35.(i) Depict the equipotential surfaces for a system of two identical positive point charges placed a distance d apart.

(ii) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points with positions r_1 and r_2 respectively, in presence of external electric field E . [Delhi 2010]

Ans.

(i) The figure is shown as below :

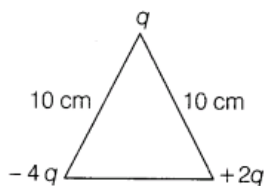


Equipotential surfaces of two identical positive charges

By definition, electric potential energy of any charge q placed in the region of electric field is equal to the work done in bringing charge q from infinity to that point and given by $U = qV$ where, V is the electric potential (as potential at infinity is assumed to be zero) where, the charge q is placed. Now, considering the electric potentials at positions r_1 and r_2 as V_1 and V_2 , respectively. Therefore, total potential energy of the system of two charges q_1 and q_2 placed at points with position vectors r_1 and r_2 in the region of E is given by $U =$ work done in bringing charge q_1 from infinity to that position in E is equal to work done for charge q_2 from infinity to that position in E + work done to that of charge q_2 at these positions in presence of q_1

$$\begin{aligned} \text{i.e. } U &= q_1 V_1 + q_2 V_2 \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|r_2 - r_1|} \end{aligned}$$

36. Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown given in the figure.



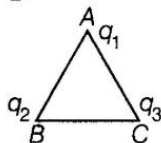
Here, $q = 1.6 \times 10^{-10} \text{ C}$. [Delhi 2008]

Ans.



The total electrostatic potential energy of this system is

$$U = \frac{1}{4\pi\epsilon_0 R} \left[\frac{q_1 q_2}{AB} + \frac{q_2 q_3}{BC} + \frac{q_3 q_1}{CA} \right]$$



Work done in making the system U = Total electrostatic potential energy of the system

Distance, $r = 10 \text{ cm} = 0.1 \text{ m}$ (1)

$U = U_{12} + U_{23} + U_{31}$ (1/2)

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q(-4q)}{r} + \frac{(-4q)(2q)}{r} + \frac{(2q)(q)}{r} \right]$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot \frac{10q^2}{r}$$

$$= -9 \times 10^9 \times \frac{10 \times (1.6 \times 10^{-10})^2}{0.1}$$

$U = -1.44 \times 10^{-8} \text{ J}$ (1/2)

\therefore Work done to dissociate the system

$$W = -U$$

$$W = +1.44 \times 10^{-8} \text{ J} \quad (1)$$

37.(i) Two point charges q_1 and q_2 initially at infinity, are brought one by one to points P_1 and P_2 specified by position vectors \mathbf{r}_1 and \mathbf{r}_2 relative to same origin. What is the potential energy of this charge configuration? Define an equipotential surface.

(ii) Draw schematically the equipotential surface corresponding to a field that uniformly increases in magnitude but remains constant in direction. [Delhi 2008]

Ans.

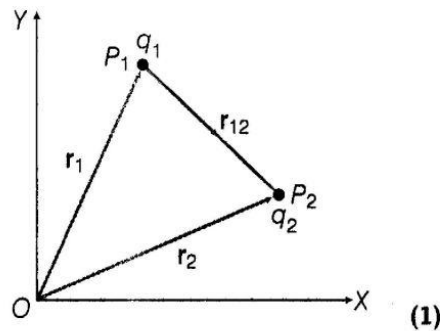


Potential energy of a system of two charges depends upon position vector of one with respect to other.

- (i) Potential energy of the combination of two charges

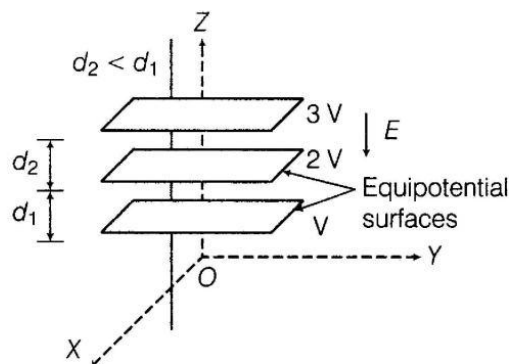
$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|r_{12}|}$$

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|r_2 - r_1|}$$



- (ii) (a) **Equipotential surface** A surface which has equal potential at every point on it, is called equipotential surface. It can be obtained by finding locus or set of points in the region of electric field which are at the same potential. (1)

- (b) The figure of equipotential surface is shown as below :

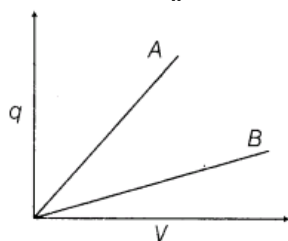


Equipotential surfaces corresponding to a field that uniformly increases in magnitude (1)

Capacitance

1 Mark Questions

1. The given graph shows the variation of charge q versus potential difference V for two capacitors C_1 and C_2 . Both the capacitors have same plate separation but plate area of C_2 is greater than that of C_1 . Which line (A or B) corresponds to and why? [All India 2014 C]



Ans. Line B corresponds to C_2 , because slope (q/v) of B is less than slope of A.

2. A capacitor has been charged by a DC source. What are the magnitude of conduction and displacement current when it is fully charged? [Delhi 2013]

Ans.

Electric flux through capacitor's plates,

$$\phi_E = \frac{q}{\epsilon_0}$$

Here, q = charge on capacitor

Displacement current,

$$I_D = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d\left(\frac{q}{\epsilon_0}\right)}{dt} = 0$$

Conduction current, $I = C \frac{dV}{dt} = 0$, as voltage becomes constant when the capacitor becomes fully charged. (1)

3. Distinguish between a dielectric and a conductor. [Delhi 2012]

Ans. Dielectrics are non-conductors and do not have free electrons at all. While conductor has free electrons in its any volume which makes it able to pass the electricity through it.

4. Define the dielectric constant of a medium. What is its unit? [Delhi 2011c]

Ans. Dielectric When a dielectric slab is introduced between the plates of charged capacitor or in the region of electric field, an electric field E_p induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence, net electric field inside the dielectric gets reduced to $E_0 - E_p$, where E_0 is external electric field. The ratio of applied external electric field and reduced electric field is known as



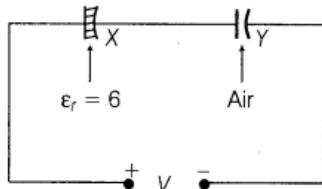
dielectric constant K of dielectric medium, i.e.

$$K = \frac{E_0}{E_0 - E_p}$$

5. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor? [Foreign 2009]

Ans. If a metal plate is introduced between the plates of a charged parallel plate capacitor. The capacitance of parallel plate capacitor will become infinite.

6. In the figure given below X, Y represent parallel plate capacitors having the same area of plates and the same distance of separation between them. What is the relation between the energies stored in the capacitors?



[Foreign 2008]

Ans.



Charge on each capacitor connected in series combination is same. Also, two capacitors having same plate area and in equal separation have equal capacitance in air or vacuum.

$$\therefore C_X = 6C_Y$$

Charge, q is same on both capacitors as connected in series combination.

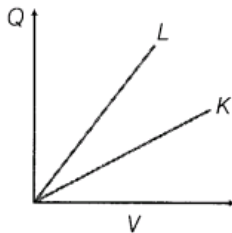
Energy stored in a capacitor is given by

$$U = \frac{q^2}{2C} \Rightarrow U \propto \frac{1}{C}$$

$$\therefore \frac{U_X}{U_Y} = \frac{C_Y}{C_X} = \frac{1}{6}$$

$$U_X = \frac{1}{6} U_Y \quad (1)$$

7. The following graph shows the variation of charge Q with voltage V for two capacitors K and L. In which capacitor is more electrostatic energy stored?



[HOTS, Foreign 2008]

Ans.

The slope of Q - V graph of capacitor gives capacitance. Therefore, capacitor L has greater capacitance. Since, two capacitors are given same voltage V , therefore energy stored in a capacitor will be given by formula.

$$U = \frac{1}{2} CV^2 \text{ and for same } V$$

$$U \propto C$$

$$\therefore C_L > C_K$$

$$\Rightarrow U_L > U_K$$

\Rightarrow Capacitor L will store more electrostatic energy. (1)

8. Define dielectric strength of a dielectric. [Delhi 2008 C]

Ans. The dielectric strength of a dielectric is the maximum value of applied electric field required to just breakdown of the dielectric material.

2 Marks Questions

9. A parallel plate capacitor of capacitance C is charged to a potential V . It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. [All India 2014]

Ans.

Let q be the charge on the charged capacitor.

\therefore Energy stored in it is given by

$$U = \frac{q^2}{2C}$$

When another uncharged similar capacitor is connected, then the net capacitance of the system is given by

$$C' = 2C \quad (1)$$

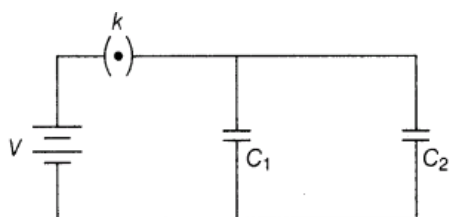
The charge on the system remains constant. So, the energy stored in the system is given by

$$U' = \frac{q^2}{2C'} = \frac{q^2}{4C} \quad [\because C' = 2C]$$

Thus, the required ratio is given by

$$\frac{U'}{U} = \frac{q^2 / 4C}{q^2 / 2C} = \frac{1}{2}$$

10. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 = 2C_2$ are connected across a battery of V volt as shown in the figure. Initially, the key (k) is kept closed to fully charge the capacitors. The key is now thrown open and a dielectric slab of dielectric constant K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of (i) the net capacitance and (ii) the energies stored in the combination before and after the introduction of the dielectric slab. [Delhi 2014 C]



Ans.

(i) Given, $C_1 = 2C_2$... (i)

Net capacitance before filling the gap with dielectric slab is given by

$$C_{\text{initial}} = C_1 + C_2 \quad [\text{from Eq. (i)}]$$

$$C_{\text{initial}} = 2C_2 + C_2 = 3C_2 \quad \dots (ii)$$

Net capacitance after filling the gap with dielectric slab of electric constant K

$$C_{\text{initial}} = KC_1 + KC_2 = K(C_1 + C_2) \quad [\text{from Eq. (ii)}]$$

$$C_{\text{final}} = 3KC_2 \quad \dots (iii)$$

Ratio of net capacitance is given by

$$\frac{C_{\text{initial}}}{C_{\text{final}}} = \frac{3C_2}{3KC_2} = \frac{1}{K} \quad [\text{from Eqs. (ii) and (iii)}] \quad \text{(1)}$$

(ii) Energy stored in the combination before introduction of dielectric slab

$$U_{\text{initial}} = \frac{Q^2}{3C_2} \quad \dots (iv)$$

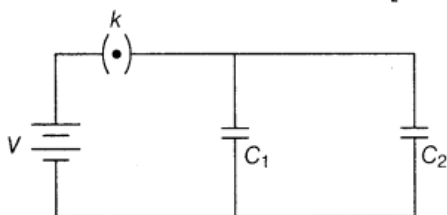
Energy stored in the combination after introduction of dielectric slab.

$$U_{\text{final}} = \frac{Q^2}{3KC_2} \quad \dots (v)$$

Ratio of energies stored

$$\frac{U_{\text{initial}}}{U_{\text{final}}} = \frac{K}{1} \quad [\text{from Eqs. (iv) and (v)}] \quad \text{(1)}$$

11. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 = 2C_2$ are connected across a battery of V volts as shown in the figure. Initially, the key (k) is kept closed to fully charge the capacitors. The key is now thrown open and a dielectric slab of dielectric constant K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of (i) the net capacitance and (ii) the energies stored in the combination before and after the introduction of the dielectric slab. [Delhi 2014 C]



Ans.

(i) Given, $C_1 = 2C_2$... (i)

Net capacitance before filling the gap with dielectric slab is given by

$$C_{\text{initial}} = C_1 + C_2 \quad [\text{from Eq. (i)}]$$

$$C_{\text{initial}} = 2C_2 + C_2 = 3C_2 \quad \dots (ii)$$

Net capacitance after filling the gap with dielectric slab of electric constant K

$$C_{\text{initial}} = KC_1 + KC_2 = K(C_1 + C_2) \quad [\text{from Eq. (ii)}]$$

$$C_{\text{final}} = 3KC_2 \quad \dots (iii)$$

Ratio of net capacitance is given by

$$\frac{C_{\text{initial}}}{C_{\text{final}}} = \frac{3C_2}{3KC_2} = \frac{1}{K} \quad [\text{from Eqs. (ii) and (iii)}] \quad \text{(1)}$$

(ii) Energy stored in the combination before introduction of dielectric slab

$$U_{\text{initial}} = \frac{Q^2}{3C_2} \quad \dots (iv)$$

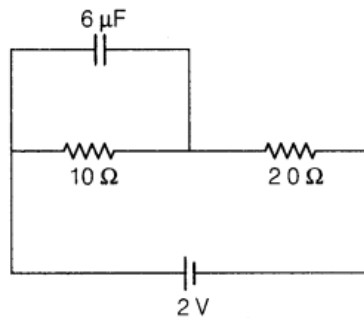
Energy stored in the combination after introduction of dielectric slab.

$$U_{\text{final}} = \frac{Q^2}{3KC_2} \quad \dots (v)$$

Ratio of energies stored

$$\frac{U_{\text{initial}}}{U_{\text{final}}} = \frac{K}{1} \quad [\text{from Eqs. (iv) and (v)}] \quad \text{(1)}$$

12. Find the charge on the capacitor as shown in the circuit. [Foreign 2014]



Ans.

Total current through the circuit is given by
 $I = V / R$

Here, $V = 2 \text{ V}$

$$R = (10 + 20) \Omega = 30 \Omega$$

$$\therefore I = \frac{2}{30} = \frac{1}{15} \text{ A}$$

Voltage across 10Ω resistor

$$= I(10) = 10/15 = \frac{2}{3} \text{ V}$$

Charge on the capacitor is given by

$$Q = CV = (6 \times 10^{-6}) \times 2/3 = 4 \mu \text{ C} \quad \text{(1)}$$

13. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor, but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. [Delhi 2013]

Ans.

Initially, when there is a vacuum between the two plates, then capacitance of the plate is $C_0 = \frac{\epsilon_0 A}{d}$, where A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced. Now, if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E .

Now, the gap between plates is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0 . (1)

If V be the potential difference between the plates of the capacitor, then $V = Et + E_0(d - t)$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} = \frac{d}{2}(E + E_0) \quad \left[\because t = \frac{d}{2} \right]$$

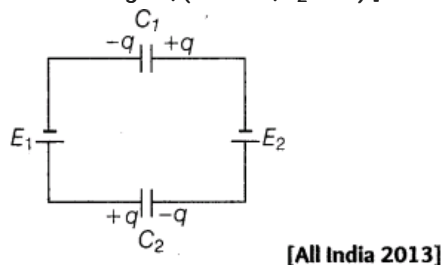
$$\Rightarrow V = \frac{d}{2} \left(\frac{E_0}{K} + E_0 \right) = \frac{dE_0}{2K} (K + 1) \quad \left[\text{As, } \frac{E_0}{E} = K \right]$$

$$\text{Now, } E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

$$\Rightarrow V = \frac{d}{2K} \cdot \frac{q}{\epsilon_0 A} (K + 1)$$

$$\text{We know that, } C = \frac{q}{V} = \frac{2K\epsilon_0 A}{d(K + 1)} \quad (1)$$

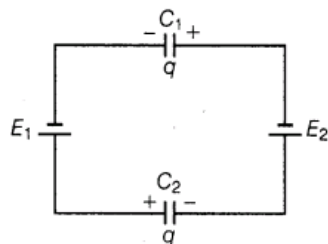
14. Determine the potential difference across the plates of the capacitor C_x of the network shown in the figure, (assume, $E_2 > E_1$) [All India 2013]



Ans.

$$\text{Potential difference, } \frac{-q}{C_1} + E_1 - \frac{q}{C_2} - E_2 = 0$$

$$\text{or } \frac{q}{C_1} + \frac{q}{C_2} = E_1 - E_2 \quad (1)$$



$$\text{Now, } V_1 = \frac{-q}{C_1} \text{ and } V_2 = \frac{+q}{C_2} \quad (1)$$

15. Net capacitance of three identical capacitors in series is $1\ \mu\text{F}$. What will be their net capacitance if connected in parallel?

Find the ratio of energy stored in the two configurations if they are both connected to the same source.

[All India 2011]

Ans.



If n identical capacitors, each of capacitance C are connected in series combination give equivalent capacitance,

$$C_s = \frac{C}{n}$$

and when connected in parallel combination then equivalent capacitance,

$$C_p = nC$$

Also, for same voltage, energy stored in the capacitor is given by

$$U = \frac{1}{2} CV^2 \quad [\text{for constant}]$$

$$U \propto C$$

$$C_s = 1\ \mu\text{F} \quad [\because n = 3]$$

In series combination, $C_s = \frac{C}{n}$

In parallel combination,

$$\Rightarrow C_p = nC$$

According to the problem,

$$C = nC_s = 3 \times 1\ \mu\text{F} = 3\ \mu\text{F}$$

For each capacitor,

In parallel combination,

$$C_p = nC = 3 \times 3 = 9\ \mu\text{F}$$

$$C_p = 9\ \mu\text{F} \quad (1)$$

For same voltage, $U \propto C \Rightarrow \frac{U_s}{U_p} = \frac{C_s}{C_p}$

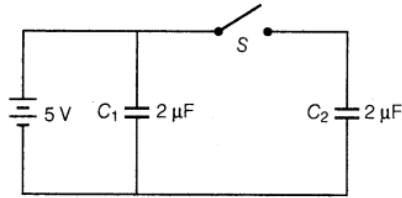
$$= \frac{C/n}{nC} = \frac{1}{n^2}$$

$$\Rightarrow \frac{1}{(3)^2} = \frac{1}{9}$$

$$\Rightarrow \frac{U_s}{U_p} = \frac{1}{9}$$

$$\text{or } U_s : U_p = 1 : 9 \quad (1)$$

16. Figure shows two identical capacitors C_1 and C_2 , each of $2\ \mu\text{F}$ capacitance, connected to a battery of $5\ \text{V}$. Initially, switch S is closed. After sometime, S is left open and dielectric slabs of dielectric constant $K = 5$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?



Ans.



When a dielectric medium of dielectric constant K is introduced,

- (i) in an isolated (not connected with battery) capacitor, then total charge on capacitor remains same.
- (ii) in a capacitor connected with battery, then potential difference across the capacitor remains same as that of potential difference across battery.

Two identical capacitors C_1 and C_2 get fully charged with $5\ \text{V}$ battery initially.

So, the charge and potential difference on both capacitors becomes

$$q = CV$$

$$= 2 \times 10^{-6} \times 5\ \text{V} = 10\ \mu\text{C}$$

and $V = 5\ \text{V}$

On introduction of dielectric medium of $K = 5$.

For C_1 (continue to be connected with battery) potential difference of C_1 , $(V') = 5\ \text{V}$

Capacitance of $C'_1 = KC = 5 \times 2\ \mu\text{F} = 10\ \mu\text{F}$

Charge, $q' = C'V' = (10\ \mu\text{F})(5\ \text{V}) = 50\ \mu\text{C}$ (1)

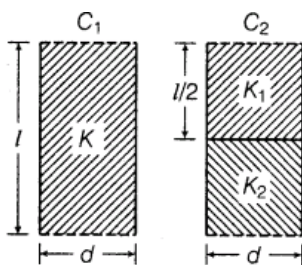
For C_2 (disconnected with battery)

Charge, $q' = q = 10\ \mu\text{C}$

Potential difference,

$$V' = \frac{V}{K} = \frac{5}{5} = 1\ \text{V} \quad (1)$$

17. Two identical parallel plate (air) capacitors C_x and C_2 have capacitance C each. The space between their plates is now filled with dielectrics as shown in the figure. If the two capacitors still have equal capacitance, they obtain the relation between dielectric constants K , K_x and K_2 . [Foreign 2011]



Ans.

⚡ The capacity of condenser is proportional to the area and inversely proportional to the distance between its plates. If a medium of dielectric constant K is filled in the space between the plates, its capacity becomes K times the capacity when there is air between the plates.

After inserting the dielectric medium, let their capacitances become C'_1 and C'_2 .

For C_1 $C'_1 = KC$... (i) (1/2)

For C_2 $C'_2 = \frac{K_1 \epsilon_0 (A/2)}{d} + \frac{K_2 \epsilon_0 A/2}{d}$

C_2 acts as if two capacitors each of area $A/2$ and separation d are connected in parallel combination.

$$C'_2 = \frac{\epsilon_0 A}{d} \left(\frac{K_1}{2} + \frac{K_2}{2} \right)$$

$$C'_2 = C \left(\frac{K_1 + K_2}{2} \right) \quad \dots (ii)$$

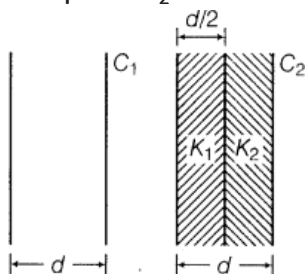
$$\left[\because C = \frac{\epsilon_0 A}{d} \right] \quad (1/2)$$

According to the problem,

$$C'_1 = C'_2$$

$$KC = C \left(\frac{K_1 + K_2}{2} \right) \Rightarrow K = \frac{K_1 + K_2}{2} \quad (1)$$

18. You are given an air filled parallel plate capacitor C_1 . The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in figure. Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d .



[Foreign 2011]

Ans.

After introduction of dielectric medium of dielectric constants K_1 and K_2 , capacitor acts as if it consists of two capacitors, each having plates of area A and separation $\frac{d}{2}$ connected in series combination for

$$C_1 = \frac{\epsilon_0 A}{d} \quad \dots(i)$$

$$\frac{1}{C_2} = \frac{1}{\left(\frac{K_1 \epsilon_0 A}{d/2}\right)} + \frac{1}{\left(\frac{K_2 \epsilon_0 A}{d/2}\right)} \quad (1)$$

$$\frac{1}{C_2} = \frac{1}{\left(\frac{\epsilon_0 A}{d}\right)} \left(\frac{1}{2K_1} + \frac{1}{2K_2}\right)$$

$$\frac{1}{C_2} = \frac{1}{2C_1} \left(\frac{K_2 + K_1}{K_1 K_2}\right)$$

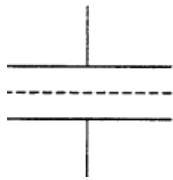
$$C_2 = C_1 \left(\frac{2K_1 K_2}{K_1 + K_2}\right) \quad (1)$$

The capacitors will be in series.

19. Figure shows a sheet of aluminium foil of negligible thickness placed between the plates of a capacitor. How will its capacitance be affected if

(i) the foil is electrically insulated?

(ii) the foil is connected to the upper plate with a conducting wire? [Foreign 2011]



Ans.

(i) The system will be equivalent to two identical capacitors connected in series combination in which two plates of each capacitor have separation half of the original separation.

Thus, new capacitance of each capacitor

$$C' = 2C \quad \left[\because C \propto \frac{1}{d} \right]$$

\therefore C and C' are in series

$$\Rightarrow C_{\text{net}} = \frac{2C \times 2C}{2C + 2C} = C$$

$$C_{\text{net}} = C$$

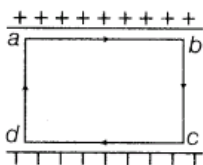
= Original capacitor (1)

(ii) System reduces to a capacitor whose separation reduces to half of original one.

\therefore New capacitance $C' = 2C$ (1)

20.(i) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(ii) The electric field inside a parallel plate capacitor is E . Find the amount of work done in moving a charge q over a closed rectangular loop $abcd$. [Delhi 2014]



Ans.(i) The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.

Let us assume that initially both the plates are uncharged. Now, we have to repeatedly remove small positive charges from one plate and transfer them to other plate.

Now, when an additional small charge (dq) is transferred from one plate to another, the small work done is given by

$$dW = V'dq = \frac{q'}{C} dq \quad (1)$$

[let charge on plate, when dq charge is transferred is q']

The total work done in transferring charge Q is given by

$$\begin{aligned} W &= \int_0^Q \frac{q'}{C} dq = \frac{1}{C} \int_0^Q q' dq \\ &= \frac{1}{C} \left[\frac{(q')^2}{2} \right]_0^Q = \frac{Q^2}{2C} \end{aligned}$$

This work is stored as electrostatic potential energy U in the capacitor.

$$U = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} \quad [\because Q = CV]$$

$$U = \frac{1}{2} CV^2 \quad (1)$$

The energy stored per unit volume of space in a capacitor is called energy density.

$$u = \frac{\frac{1}{2} CV^2}{Ad}$$

$$u = \frac{1}{2} \frac{\epsilon_0 AV^2}{d^2 A}$$

Energy density,

$$u = \frac{1}{2} \epsilon_0 E^2$$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitor.

i.e. $U = U_1 + U_2 + U_3 + \dots$

- (ii) Due to conservative nature of electric force, the work done in moving a charge in a close path in a uniform electric field is zero. (1)

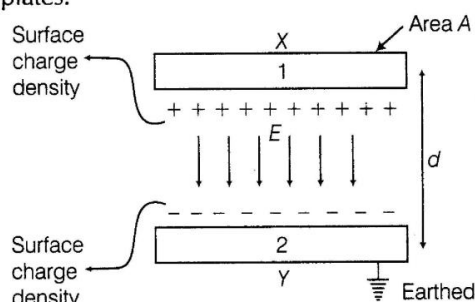
3 Marks Questions

21.(i) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d .

(ii) Two charged spherical conductors of radii r_1 and r_2 when connected by a conducting plate respectively. Find the ratio of their surface charge densities in terms of their radii. [Delhi 2014]

Ans.

- (i) Parallel plate capacitor consists of two thin conducting plates each of area A held parallel to each other at a suitable distance d . One of the plates is insulated and other is earthed. There is a vacuum between the plates.



(1)

Suppose, the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges. Suppose, the surface density of charge on each plate is σ . We know that the intensity of electric field at a point between two plane parallel sheets of equal and opposite charges is σ / ϵ_0 , where ϵ_0 is the permittivity of free space.

The intensity of electric field between the plates will be given by, $E = \frac{\sigma}{\epsilon_0}$

The charge on each plate is q and the area of each plate is A . Thus,

$$\sigma = \frac{q}{A} \text{ and } E = \frac{q}{\epsilon_0 A} \quad \dots(i)$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed$$

Substituting the value of E from Eq. (i), we get

$$V = \frac{qd}{\epsilon_0 A}$$

∴ Capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{qd/\epsilon_0 A} \quad \text{or} \quad C = \frac{\epsilon_0 A}{d}$$

where, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

It is clear from this formula that in order to obtain high capacitance

(a) A should be large, i.e. the plates of large area should be taken.

(b) d should be small, i.e. the plates should be kept closer to each other. (1)

(ii) Surface charge density is given by

$$\sigma = \frac{q}{4\pi R^2}$$

After connecting both the conductors, their potentials will become equal.

$$V_1 = V_2 \\ \Rightarrow \frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \quad [\because \text{for spherical conductors}]$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R} \quad \text{or} \quad V = \frac{KV}{R}$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{q_1 / 4\pi R_1^2}{q_2 / 4\pi R_2^2}$$

$$= \frac{q_1}{q_2} \left(\frac{R_2}{R_1} \right)^2 = \frac{R_2}{R_1}$$

22. In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \text{ m}^2$ and the separation between the plate is 3 mm.

(i) Calculate the capacitance of the capacitor.

(ii) If this capacitor is connected to 100 V supply, what would be the change on each plate?

(iii) How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected? [Foreign 2014]

Ans.

Given, area of each plate, $A = 6 \times 10^{-3} \text{ m}^2$

Distance between plates d ,

$$= 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

(i) Capacitance of parallel plate capacitor is given by

$$C = \frac{\epsilon_0 A}{d}$$

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$$C = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$$C = 1.77 \times 10^{-11} \text{ F} \quad (1)$$

(ii) Charge on parallel plate capacitor is given by

$$Q = CV$$

Given, $V = 100 \text{ V}$

Now,

$$Q = 17.7 \times 10^{-12} \times 100$$

$$Q = 1.77 \times 10^{-10} \text{ C} \quad \dots(i) \quad (1)$$

(iii) Given, $K = 6$

$$\text{Now, } C' = KC$$

$$\therefore Q' = KQ \quad [\text{From Eq. (i)}]$$

$$Q' = 6 \times 1.77 \times 10^{-10}$$

$$Q' = 10.62 \times 10^{-10} \text{ C} \quad [1]$$

23. A capacitor of unknown capacitance is connected across a battery of V volt. The charge stored in it is $360 \mu\text{C}$. When potential across the capacitor is reduced by 120 V , the charge stored in it becomes $120 \mu\text{C}$. Calculate

(i) the potential V and the unknown capacitance C .

(ii) what will be the charge stored in the capacitor if the voltage applied had increased by 120 V ? [Delhi 2000]

Ans.

(i) We have initial voltage, $V_1 = V$ volt and charge stored, $Q_1 = 360 \mu\text{C}$.

$$Q_1 = CV_1 \quad \dots(i)$$

Charged potential, $V_2 = V - 120$

$$Q_2 = 120 \mu\text{C} \quad (1/2)$$

$$Q_2 = CV_2 \quad \dots(ii)$$

By dividing Eq. (ii) from Eq. (i), we get

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2}$$

$$\Rightarrow \frac{360}{120} = \frac{V}{V - 120}$$

$$\Rightarrow V = 180 \text{ V} \quad (1)$$

$$\therefore C = \frac{Q_1}{V_1} = \frac{360 \times 10^{-6}}{180}$$

$$= 2 \times 10^{-6} \text{ F} = 2 \mu\text{F}$$

Hence, the potential, $V = 180 \text{ V}$ and unknown capacitance is $2 \mu\text{F}$. (1)

(ii) Let Q be the charge stored in the capacitor

$$Q = CV$$

$$= 2 \times 10^{-6} \times 120$$

$$Q = 24 \times 10^{-5} \text{ C} \quad (1/2)$$

24. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF . Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. [Foreign 2012]

Ans.

Given, $C = 200 \text{ pF} = 200 \times 10^{-12} \text{ F}$ and $V = 300 \text{ V}$

The energy (initially) stored by the capacitor is

$$\begin{aligned} U_i &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} \times 200 \times 10^{-12} \times 300 \times 300 \\ &= 9 \times 10^{-6} \text{ J} \end{aligned}$$

The charge on the capacitor when charge through 300 V battery is

$$\begin{aligned} Q &= CV \\ &= 200 \times 10^{-12} \times 300 \\ &= 6 \times 10^{-8} \text{ C} \end{aligned} \quad (1)$$

When two capacitors are connected, they have their positive plates at the same potential and negative plates also at the same potential. Let V be the common potential difference. By charge conservation, charge would distribute but total charge would remain constant.

$$\begin{aligned} \text{Thus, } Q &= q + q' \\ \frac{q}{C} &= \frac{q'}{C'} \\ \frac{q}{200} &= \frac{q'}{100} \\ q &= 2q' \end{aligned} \quad (1)$$

$$\text{Thus, } Q = 2q' + q' = 3q'$$

$$\begin{aligned} \text{So, } q' &= \frac{Q}{3} = \frac{60 \text{ nC}}{3} \\ &= 20 \text{ nC} \end{aligned}$$

$$\text{and } q = 2q' = 40 \text{ nC}$$

Thus, final energy

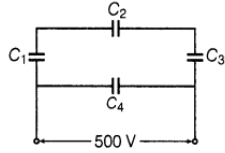
$$\begin{aligned} U_f &= \frac{q^2}{2C} + \frac{q'^2}{2C'} \\ &= \frac{1}{2} \times \frac{(40 \times 10^{-9})^2}{200 \times 10^{-12}} + \frac{1}{2} \times \frac{(20 \times 10^{-9})^2}{100 \times 10^{-12}} \\ &= 4 \times 10^{-6} + 2 \times 10^{-6}, \\ &= 6 \times 10^{-6} \text{ J} \end{aligned}$$

Difference in energy

$$\begin{aligned} &= \text{final energy} - \text{initial energy} \\ &= U_f - U_i \\ &= 6 \times 10^{-6} - 9 \times 10^{-6} \\ &= -3 \times 10^{-6} \end{aligned} \quad (1)$$

Thus, difference in energy is $-3 \times 10^{-6} \text{ J}$.

25. A network of four capacitors each of $12\ \mu\text{F}$ capacitance is connected to a 500 V supply as shown in the figure. Determine



- (i) the equivalent capacitance of the network and
(ii) the charge on each capacitor.

[HOTS; All India 2010]

Ans.

💡 In a series combination, where there is no division of charge,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

In a parallel combination, where potential difference is same.

$$C_p = C_1 + C_2 + C_3 \dots$$

- (i) Here, C_1 , C_2 and C_3 are in series; therefore their equivalent capacitance

$$\begin{aligned} \frac{1}{C'} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ C' &= \frac{C}{3} = \frac{12}{3} = 4\ \mu\text{F} \end{aligned} \quad (1)$$

Now, C' and C are in parallel combination.

$$\begin{aligned} \therefore C_{\text{net}} &= C' + C \\ &= 4\ \mu\text{F} + 12\ \mu\text{F} = 16\ \mu\text{F} \\ C_{\text{net}} &= 16\ \mu\text{F} \end{aligned} \quad (1)$$

- (ii) Being C' and C in parallel, 500 V potential difference is applied across them.

\therefore Charge on C'

$$\begin{aligned} q_1 &= C'V \\ &= (4\ \mu\text{F}) \times 500 \\ &= 2000\ \mu\text{C} \end{aligned}$$

$\therefore C_1, C_2$ and C_3 capacitors each will have $2000\ \mu\text{C}$ charge.

$$\begin{aligned} \text{Charge on } C_4, q_2 &= C \times V \\ &= 12 \times 500 \\ &= 6000\ \mu\text{C} \end{aligned} \quad (1)$$

26. A parallel plate capacitor is charged by a battery. After sometime, the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will

- (i) the capacitances of the capacitor,
(ii) potential difference between the plates and
(iii) the energy stored in the capacitors be affected? Justify your answer in each case. [Delhi 2010]

Ans.

On introduction of dielectric slab in a isolated charged capacitor.

- (i) The capacitance (C') becomes K times of original capacitor as

$$C = \frac{\epsilon_0 A}{d}$$

and $C' = \frac{K \epsilon_0 A}{d} \quad (1)$

- (ii) \therefore Charge remains conserved in the phenomenon.

$$\therefore CV = C'V'$$

$$V' = \frac{CV}{C'} = \frac{CV}{KC} \quad [\text{refer part (i)}]$$

$$\Rightarrow V' = \frac{V}{K} \quad (1)$$

Potential difference decreases and become $\frac{1}{K}$ times of original value.

- (iii) Energy stored initially,

$$U = \frac{q^2}{2C}$$

Energy stored later,

$$\therefore U' = \frac{q^2}{2(KC)} \quad [\because C' = KC]$$

where, K = dielectric constant of medium

$$\Rightarrow U' = \frac{1}{K} \left(\frac{q^2}{2C} \right)$$

$$\Rightarrow U' = \frac{1}{K} (U)$$

$$U' = \frac{1}{K} \times U$$

The energy stored in the capacitor decreases and becomes $\frac{1}{K}$ times of original energy. (1)

27. A parallel plate capacitor, each with plate area A and separation d is charged to a potential difference V . The battery used to charge it remains connected. A dielectric slab of thickness d and dielectric constant K is now placed between the plates. What change if any will take place in

(i) charge on plates?

(ii) electric field intensity between the plates?

(iii) capacitance of the capacitor? Justify your answer in each case. [Delhi 2010]

Ans.

On introduction of dielectric slab to fill the gap between plates of capacitor completely when capacitor is connected with battery.

- (i) The potential difference V between capacitors is same due to connectivity with battery and hence, charge q' becomes K times of original charge as

$$q' = C' V' = (KC) (V) = K(CV) = Kq$$

$$q' = Kq \quad (1)$$

- (ii) Electric field intensity continue to be the same as potential difference and separation between two plates remain unaffected as

$$E = \frac{V}{d} \quad (1)$$

- (iii) The capacitance of capacitor becomes K times of original capacitor.

$$\therefore C' = KC = \frac{K \epsilon_0 A}{d} \quad (1)$$

28. A parallel plate capacitor is charged to a potential difference V by a DC source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason, how the following will change?

- (i) Electric field between the plates
(ii) Capacitance
(iii) Energy stored in the capacitor [Delhi 2010]

Ans.

After disconnection from battery and doubling the separation between two plates

- (i) Charge on capacitor remains same.

$$\text{i.e. } CV = C' V'$$

$$\Rightarrow CV = \left(\frac{C}{2}\right) V' \Rightarrow V' = 2V$$

∴ Electric field between the plates

$$E' = \frac{V'}{d'} = \frac{2V}{2d}$$

$$E' = \frac{V}{d} = E$$

⇒ Electric field between the two plates remains same. (1)

(ii) Capacitance reduces to half of original value as

$$C \propto \frac{1}{d} \Rightarrow C' = \frac{C}{2} \quad (1)$$

(iii) Energy stored in the capacitor before disconnection from battery

$$U_1 = \frac{q^2}{2C}$$

Now, energy stored in the capacitor after disconnection from battery

$$U_2 = \frac{q^2}{2(C')} = \frac{q^2}{2 \times \left(\frac{C}{2}\right)} = \frac{q^2}{C}$$

$$\Rightarrow U_2 = 2 \left(\frac{q^2}{2C} \right) = 2U_1$$

$$U_2 = 2U_1$$

Energy stored in capacitor gets doubled to its initial value. (1)

29. Find the ratio of the potential differences that must be applied across the parallel and the series combination of two identical capacitors so that the energy stored in the two cases becomes the same.

[Foreign 2010]

Ans.

Let V_1 and V_2 are the potential differences across the series and parallel combination of two identical capacitors each of capacitance C .

Equivalent capacitance in series combination

$$C_s = \frac{C}{2} \quad (1)$$

Equivalent capacitance in parallel combination

$$C_p = 2C$$

According to the question,

$$U_s = U_p \quad (1)$$

$$\frac{1}{2} C_s V_s^2 = \frac{1}{2} C_p V_p^2$$

$$\Rightarrow \frac{V_s^2}{V_p^2} = \frac{C_p}{C_s} = \frac{2C}{\left(\frac{C}{2}\right)}$$

$$\frac{V_s^2}{V_p^2} = 4 \Rightarrow \frac{V_s}{V_p} = 2$$

$$V_s : V_p = 2 : 1$$

30.(i) How is the electric field due to a charged parallel plate capacitor affected when a

dielectric slab is inserted between the plates fully occupying the intervening region?

(ii) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has thickness $1/2 d$,

where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates. [Foreign 2010]

Ans.

- (i) The total charge on the capacitor remains conserved on introduction of dielectric slab. Also, the capacitance of capacitor increases to K times of original values.

$$\begin{aligned}\therefore CV &= C'V' \\ CV &= (KC)V' \\ \Rightarrow V' &= \frac{V}{K}\end{aligned}$$

\therefore New electric field,

$$E' = \frac{V'}{d} = \left(\frac{V/K}{d}\right) = \left(\frac{V}{d}\right) \frac{1}{K} = \frac{E}{K}$$

\therefore On introduction of dielectric medium new electric field E' becomes $\frac{1}{K}$ times of its original value. (1)

- (ii) \therefore Capacitance of a parallel plate capacitor partially filled with dielectric medium is given by

$$C = \frac{\epsilon_0 A}{(d - t + t/K)} \quad (1)$$

where, t is the thickness of dielectric medium.

Here, $t = \frac{d}{2}$

$$C = \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\epsilon_0 A}{\frac{d}{2} \left(1 + \frac{1}{K}\right)}$$

$$\therefore C = \frac{2\epsilon_0 AK}{(K+1)d} \quad (1)$$

31.(i) Plot a graph comparing the variation of potential V and electric field E due to a point charge Q as a function of distance R from the point charge.

(ii) Find the ratio of the potential differences that must be applied across the parallel and the series

combination of two capacitors, C_1 and C_2 with their capacitances in the ratio $1 : 2$, so that the energy stored in the two cases becomes the same [Foreign 2010]

Ans.

- (i) The graph comparing the variation of potential V and electric field. Refer to ans. 20 Topic -1 (1)

- (ii) Let $C_1 = C$ and $C_2 = 2C$

\therefore Equivalent capacitance

$$\text{In series, } C_s = \frac{2C \times C}{2C + C} = \frac{2C}{3}$$

$$\text{In parallel, } C_p = 2C + C = 3C \quad (1)$$

$\therefore V_p$ and V_s are potential difference across the final capacitor in parallel and series combination respectively, to have same potential energy.

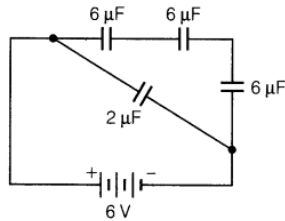
$$U_p = U_s$$

$$\frac{1}{2} C_p V_p^2 = \frac{1}{2} C_s V_s^2$$

$$\frac{V_p}{V_s} = \sqrt{\frac{C_s}{C_p}} = \sqrt{\frac{(2C/3)}{(3C)}} = \sqrt{\frac{2}{9}}$$


$$V_p : V_s = \sqrt{2} : 3 \quad (1)$$

32. Four capacitors of values $6 \mu\text{F}$, $6 \mu\text{F}$, $6 \mu\text{F}$ and $2 \mu\text{F}$ are connected to a 6 V battery as shown in the figure. Determine the



- (i) Equivalent capacitance of the network.
(ii) The charge on each capacitor.
[Delhi 2010C]

Ans.

 In series combination, charge on each capacitor is the same.

- (i) All capacitors of $6 \mu\text{F}$ are in series combination, then equivalent capacitance is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

or $C' = \frac{C}{n} = \frac{6 \mu\text{F}}{3} = 2 \mu\text{F}$

C' and $2 \mu\text{F}$ capacitors are in parallel combination.

\therefore Equivalent capacitance

$$C_{\text{eq}} = C' + 2 \mu\text{F}$$

$$= 2 \mu\text{F} + 2 \mu\text{F}$$

$$C_{\text{eq}} = 4 \mu\text{F} \quad (1)$$

- (ii) Since, C' and $2 \mu\text{F}$ are in parallel combination, therefore, same potential difference 6 V is applied on them.

\therefore Charge on C'

$$q' = C' V = (2 \mu\text{F}) \times 6 \text{ V} = 12 \mu\text{C}$$

The charge across the each capacitor of $6 \mu\text{F}$ capacitor is same and equal to charge across the combination i.e. $12 \mu\text{C}$.

Charge on $2 \mu\text{F}$ capacitor

$$q = CV = (2 \mu\text{F}) (6 \text{ V}) = 12 \mu\text{C}$$

33. A parallel plate* capacitor is charged by a battery. After sometime, the battery is disconnected and a dielectric slab of dielectric constant K is inserted between the plates.

How would

(i) the electric field between the plates

(ii) the energy stored in the capacitor be affected? Justify your answer. [All India 2009]

Ans. (i) The total charge on the capacitor remains conserved on introduction of dielectric slab.

Also, the capacitance of capacitor increases to K times of original values.

(ii) Energy stored in capacitor initially,

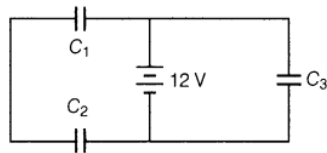
$$U = \frac{1}{2} CV^2$$

(Energy stored in capacitor later,

$$U' = \frac{1}{2} C' V'^2 = \frac{1}{2} (KC) \left(\frac{V}{K} \right)^2$$

$$U' = \frac{1}{K} \left(\frac{1}{2} CV^2 \right) = \frac{U}{K} \quad U' = \frac{U}{K}$$

34. Three identical capacitors C_1 , C_2 and C_3 of capacitances $6 \mu\text{F}$ each are connected to a 12 V battery as shown below:



Find

- (i) the charge on each capacitor
- (ii) the equivalent capacitances of the network
- (iii) the energy stored in the network of capacitors

[Delhi 2009C]

Ans.

- (i) The equivalent capacitance of C_1 and C_2 connected in series

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow C' = \frac{6}{2} = 3 \mu\text{F}$$

$$\therefore \text{Charge, } q' = C'V = (3 \mu\text{F}) 12 = 36 \mu\text{C}$$

\therefore Charge on each capacitor of C_1 and C_2 is $36 \mu\text{C}$.

Charge on C_3 ,

$$q_3 = C_3V = (6 \mu\text{F}) \times 12 = 72 \mu\text{C} \quad (1)$$

$$q_3 = 72 \mu\text{C}$$

- (ii) Equivalent capacitance of network,

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$= \frac{6 \times 6}{6 + 6} + 6$$

$$= 3 + 6 = 9 \mu\text{F}$$

$$C_{\text{eq}} = 9 \mu\text{F} \quad (1)$$

- (iii) Energy stored in the capacitor

$$U = U_1 + U_2 + U_3$$

$$= \frac{q'^2}{2C_1} + \frac{q'^2}{2C_2} + \frac{q^2}{2C_3}$$

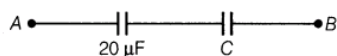
$$\therefore C_1 = C_2 = C_3 = 6 \mu\text{F}$$

$$\therefore U = \frac{1}{(12 \mu\text{F})} [q'^2 + q'^2 + q^2]$$

$$= \frac{1}{(12 \mu\text{F})} [(36 \mu\text{C})^2 + (36 \mu\text{C})^2 + (72 \mu\text{C})^2]$$

$$U = 648 \mu\text{J} \quad (1)$$

35. The equivalent capacitance of the combination between points A and B in the given figure is $4 \mu\text{F}$.



- (i) Calculate the capacitance of the capacitor C.
- (ii) Calculate the charge on each capacitor if a 12 V battery is connected across terminals A and B.
- (iii) What will be the potential drop across each capacitor?

[Delhi 2009]

Ans.

According to the question,
Capacitors of $20 \mu\text{F}$ and C are connected in series.

- (i) The equivalent capacitance

$$(4 \mu\text{F}) = \frac{(20 \mu\text{F}) \times C}{20 + C}$$

$$(20 + C) = 5C$$

$$4C = 20 \therefore C = 5 \mu\text{F} \quad (1)$$

(ii) Charge on capacitor (equivalent)

$$q = (4 \mu\text{F}) \times 12 = 48 \mu\text{C}$$

same charge $48 \mu\text{C}$ lies on both the capacitors. (1)

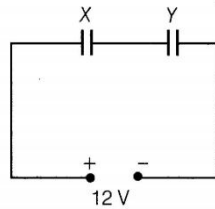
(iii) Potential drop across $20 \mu\text{F}$ capacitor

$$V_1 = \frac{q}{C_1} = \frac{48 \mu\text{C}}{20 \mu\text{F}} = 2.4 \text{ V}$$

Potential drop across $5 \mu\text{F}$ capacitor

$$V_2 = \frac{q}{C_2} = \frac{48 \mu\text{C}}{5 \mu\text{F}} = 9.6 \text{ V} \quad \left(\frac{1}{2} \times 2 = 1 \right)$$

36. Two parallel plate capacitors X and Y have the same area of the plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\epsilon_r = 4$.



- (i) Calculate the capacitance of each capacitor if equivalent of the combination is $4 \mu\text{F}$.
- (ii) Calculate the potential difference between the plates of X and Y.
- (iii) What is the ratio of electrostatic energy stored in X and Y? [Delhi 2009]

Ans.



A dielectric medium is inserted between the plates of a condenser in place of air, its capacity becomes K times of original one.

The capacitance of two capacitors are

$$C_x = \frac{\epsilon_0 A}{d} = C \quad [\text{say}]$$

$$C_y = \frac{4\epsilon_0 A}{d} = 4C$$

(i) According to the problem,

$$C_{\text{eq}} = 4\mu\text{F}$$

$$\therefore C_{\text{eq}} = \frac{C_x \times C_y}{C_x + C_y}$$

$$4\mu\text{F} = \frac{C \times 4C}{C + 4C} = \frac{4C}{5}$$

$$\left[\because \text{For series combination, } C = \frac{C_1 C_2}{C_1 + C_2} \right]$$

$$C = 5\mu\text{F}$$

$$\Rightarrow C_x = C = 5\mu\text{F}$$

$$C_y = 4C = 4 \times 5\mu\text{F} = 20\mu\text{F} \quad (1/2 \times 2 = 1)$$

(ii) \therefore Charge across the combination

$$\begin{aligned} q &= C_{\text{eq}} V \\ &= (4\mu\text{F}) \times 12 \\ &= 48\mu\text{C} \end{aligned}$$

\therefore Same charge, i.e. $48\mu\text{C}$ lies on each capacitor being in series combination.

\therefore Potential difference across C_x ,

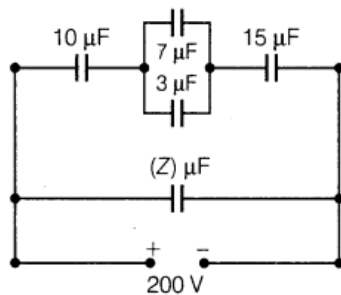
$$V_1 = \frac{q}{C_x} = \frac{48\mu\text{C}}{5\mu\text{F}} = 9.6\text{ V}$$

Potential difference across C_y ,

$$V_2 = \frac{q}{C_y} = \frac{48\mu\text{C}}{20\mu\text{F}} = 2.4\text{ V} \quad (1/2 \times 2 = 1)$$

37. A system of capacitors connected as shown in the figure has a total energy of 160 mJ stored in it. Obtain the value of the equivalent capacitance of this system and the value of Z .

[All India 2009 c]



Ans.

Given, total energy = 160 mJ

Let equivalent capacitance of the combination of capacitors is C.

According to the question,

$$\frac{1}{2} CV^2 = 160 \times 10^{-3} \text{ J}$$

$$\Rightarrow \frac{1}{2} \times C \times (200)^2 = 160 \times 10^{-3}$$

$$\Rightarrow \text{Equivalent capacitance } C = 8 \mu\text{F} \quad (1)$$

\therefore Equivalent capacitor of $7 \mu\text{F}$ and $3 \mu\text{F}$ connected in parallel = $7 + 3 = 10 \mu\text{F}$

\therefore $10 \mu\text{F}$, $10 \mu\text{F}$ [combination of $7 \mu\text{F}$ and $3 \mu\text{F}$] and $15 \mu\text{F}$ are in series combination.

Therefore, their equivalent capacitance

$$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{15} = \frac{3 + 3 + 2}{30} = \frac{8}{30}$$

$$C' = \frac{15}{4} \mu\text{F} \quad (1)$$

But, C' and Z are in parallel combination, therefore equivalent capacitance

$$\frac{15}{4} + Z = C \quad [\text{from Eq.(i)}]$$

$$\frac{15}{4} + Z = 8$$

$$Z = 8 - \frac{15}{4} = \frac{32 - 15}{4}$$

$$Z = \frac{17}{4} \mu\text{F}$$

$$Z = 4.25 \mu\text{F} \quad (1)$$

38. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes $(2/3)$ rd of its original value. What is the new distance between the plates? [All India 2008 C]
Ans.

Before introduction of slab,
 $d_1 = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

Plate area = A [say]

$$\therefore \text{Capacitance, } C_1 = \frac{\epsilon_0 A}{d_1}$$

Suppose after introduction of slab, the new distance between the plates is d_2 .

Plate area of each plate = A

$$K = 3$$

Thickness of slab,

$$t = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$\text{Capacitance, } C_2 = \frac{\epsilon_0 A}{(d_2 - t + t/K)} \quad (1)$$

According to the question,

$$\frac{2}{3} C_1 = C_2$$

$$\frac{2}{3} \cdot \frac{\epsilon_0 A}{d_1} = \frac{\epsilon_0 A}{d_2 - t + t/K}$$

Substituting the values [given in question]

$$\frac{2}{3d_1} = \frac{1}{d_2 - t + t/K} \quad (1)$$

$$\frac{2}{3 \times 4 \times 10^{-3}} = \frac{1}{(d_2 - 3 + 3/3) \times 10^{-3}}$$

[d_2 is in mm]

$$\frac{1}{6} = \frac{1}{(d_2 - 2)}$$

$$\Rightarrow d_2 - 2 = 6$$

$$\Rightarrow d_2 = 8 \text{ mm} \quad (1)$$

5 Marks Questions

39.(i) A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and a dielectric slab is inserted to completely fill the space between the plates.

How will

(a) its capacitance

(b) electric field between the plates and

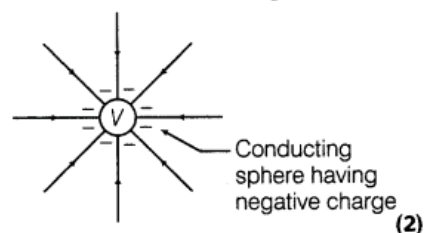
(c) energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expressions for each case.

(ii) (a) Draw the electric field lines due to a conducting sphere.

(b) Draw the electric field lines due to a dipole.

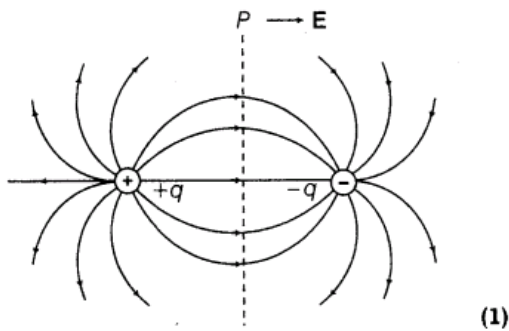
Ans.

(i) (a) Electric field lines due to a conducting sphere are shown in the figure.



(2)

(b) Electric field lines due to an electric dipole are shown in the figure



40. (i) Deduce the expression for the energy stored in a charged capacitor.

(ii) Show that the effective capacitances C of a series combination of three capacitors C_1 , C_2 and C_3 is given by

$$C = \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

[HOTS, All India 2010C]

Ans.



As, charge on capacitor increases we have to work more against electrostatic repulsion and this amount of work done will be stored as potential energy in the capacitor.

(i) We know that,

$$q = CV \Rightarrow V = q/C$$

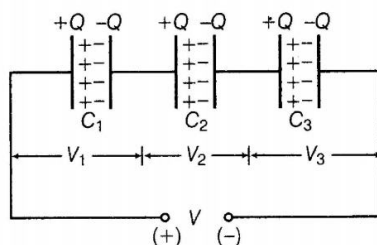
$$dW = Vdq = \frac{q}{C} dq$$

where, q = instantaneous charge,
 C = instantaneous capacitance and
 V = instantaneous voltage

\therefore Total work done in storing charge from 0 to q is given by

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C} \quad (2)$$

(ii) In series combination of capacitors, same charge lie on each capacitor for any value of capacitances.



Capacitors in series combination

(1)

Also, potential difference across the combination is equal to the algebraic sum of potential differences across each capacitor.

$$\text{i.e. } V = V_1 + V_2 + V_3 \quad \dots(i)$$

where, V_1, V_2, V_3 and V are the potential differences across C_1, C_2, C_3 and equivalent capacitor, respectively.

$$\therefore q = C_1 V_1 \Rightarrow V_1 = \frac{q}{C_1}$$

$$\text{Similarly, } V_2 = \frac{q}{C_2}$$

$$V_3 = \frac{q}{C_3} \quad [\text{from Eq. (i)}]$$

\therefore Total potential difference

$$\begin{aligned} V &= \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \\ \frac{V}{q} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ \frac{1}{C} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \end{aligned} \quad (1)$$

$[\because \frac{V}{q} = \frac{1}{C}, \text{ where } C \text{ is equivalent capacitance of combination}]$

$$\begin{aligned} \text{or } \frac{1}{C} &= \frac{C_2 C_3 + C_3 C_1 + C_1 C_2}{C_1 C_2 C_3} \\ \Rightarrow C &= \frac{C_1 C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1} \end{aligned} \quad (1)$$

41.(i) Show that in a parallel plate capacitor if the medium between the plates of a capacitor is filled with an insulating substance of dielectric constant K , its capacitance increases.

(ii) Deduce the expression for the energy stored in a capacitor of capacitance C with charge Q . [Delhi 2009 C]

Ans.

On introduction of dielectric slab in a isolated charged capacitor.

(i) The capacitance (C') becomes K times of original capacitor as

$$C = \frac{\epsilon_0 A}{d}$$

$$\text{and } C' = \frac{K \epsilon_0 A}{d} \quad (1)$$

(ii) \therefore Charge remains conserved in the phenomenon.

$$\therefore CV = C'V'$$

$$V' = \frac{CV}{C'} = \frac{CV}{KC} \quad [\text{refer part (i)}]$$

$$\Rightarrow V' = \frac{V}{K} \quad (1)$$

Potential difference decreases and become $\frac{1}{K}$ times of original value.

(iii) Energy stored initially,

$$U = \frac{q^2}{2C}$$

Energy stored later,

$$\therefore U' = \frac{q^2}{2(KC)} \quad [\because C' = KC]$$

where, K = dielectric constant of medium

$$\Rightarrow U' = \frac{1}{K} \left(\frac{q^2}{2C} \right)$$

$$\Rightarrow U' = \frac{1}{K} (U)$$

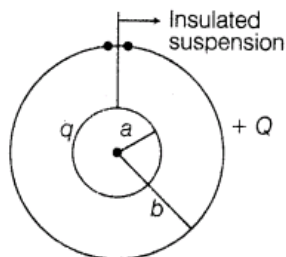
$$U' = \frac{1}{K} \times U$$

The energy stored in the capacitor decreases and becomes $\frac{1}{K}$ times of original energy. (1)

42. A small sphere of radius a carrying a positive charge q is placed concentrically inside a larger hollow conducting shell of radius b ($b > a$). This outer shell has charge Q on it. Show that if these spheres are connected by a conducting wire, charge will always flow from the inner sphere to the outer sphere irrespective of the magnitude of the two charges.

Ans.

(i) Let small sphere has charge q and radius a is placed inside a outer shell of charge $+Q$ and radius b .



Electric potential on the small sphere due to its own charge q .

$$V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a} \quad \dots(i)$$

where, q = charge on small sphere

a = radius of small sphere

Similarly, electric potential on outer sphere due to its own charge

$$V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{b} \quad \dots(\text{ii})$$

where, Q = charge of outer shell

b = radius of outer shell. $(1 \times 2 = 2)$

Also, same potential V_2 exists at every point inside outer shell due to its own charge, $+Q$.

Now, net electric potential at inner sphere of radius a .

V_i = Electric potential due to its own charge

$$V_i = \frac{1}{4\pi\epsilon_0} \frac{q}{a} + \frac{1}{4\pi\epsilon_0} \frac{Q}{b} \quad \dots(\text{iii})$$

Net electric potential at outer sphere due to charge on the both spheres

$$V_o = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{b} + \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{b} \quad \dots(\text{iv})$$

$$\therefore V_i - V_o = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) \quad \dots(\text{v})$$

From Eqs. (iii) and (iv), we get

$$\therefore a < b, \frac{1}{a} > \frac{1}{b}$$

$$\Rightarrow V_i - V_o > 0 \quad (2)$$

Thus, inner sphere has net potential higher than potential of outer sphere for every value of q and Q . Therefore, when they are connected by a wire, positive charge always flow from inner sphere (at higher potential) to outer sphere (at lower potential) irrespective of the magnitude of charge. (1)

43. (i) Derive an expression for the energy stored in a parallel plate capacitor.

(ii) On charging a parallel plate capacitor to a potential V , the spacing between the plates is halved and a dielectric medium of $\epsilon_r = 10$ is introduced between the plates without disconnecting the DC source. Explain, using suitable expressions, how the

(a) capacitance

(b) electric field and

(c) energy density of the capacitor changes? [All India 2008]

Ans.(i) An expression for energy stored in a charged capacitor. (ii) Connected battery ensures same potential difference (V) across capacitor even after reducing the spacing between plates halved and introducing the dielectric slab.

(a) If initial capacitance,

$$C_1 = \frac{\epsilon_0 A}{d}$$

Then, final capacitance

$$\begin{aligned} C_2 &= \frac{10\epsilon_0 A}{d/2} \\ &= \frac{20\epsilon_0 A}{d} = 20C_1 \end{aligned}$$

$$C_2 = 20C_1$$

Capacitance increases to 20 times of original value. (1)

(b) \therefore Initial electric field, $E_1 = \frac{V}{d}$

Finally,

$$E_2 = \frac{V}{d/2} = 2 \left(\frac{V}{d} \right) = 2E_1$$

$$E_2 = 2E_1$$

Electric field intensity gets doubled. (1)

(iii) Energy density of the capacitor

$$U_1 = \frac{1}{2} \epsilon_0 E_1^2 = \frac{1}{2} \epsilon_0 E_2^2$$

$$\begin{aligned} U_2 &= \frac{1}{2} \epsilon_0 (2E_1)^2 \\ &= \frac{1}{2} \epsilon_0 (4E_1^2) \end{aligned}$$

$$\Rightarrow U_2 = 4U_1 \quad (1)$$

Energy density becomes 4 times of its original value.

44. Define the terms (i) capacitance of a capacitor (ii) dielectric strength of a dielectric (iii) When a dielectric is inserted between the plates of a charged parallel plate capacitor fully occupying the intervening region, how does the polarisation of the dielectric medium affect the net electric field? For a linear dielectric, show that the introduction of the dielectric increases its capacitance by a factor K which is a characteristic of the dielectric. [Delhi 2008 C]

Ans.

- (i) For a given capacitor, the charge q on the capacitor is proportional to the potential difference V between the plates.

i.e. $q \propto V$
 $q = CV$ (1)

The proportionality constant, C is called the capacitance of the capacitor.

It depends on the shape, size and geometry of capacitor. (1)

- (ii) **Dielectric strength** The dielectric strength of the dielectrics is equal to that maximum value of electric field that can exist in that dielectric without causing the dielectric breakdown of its insulating property. (1)

- (iii) Other part of question refer to ans. 20(a) (topic-1) and on introduction of dielectric between the plates of capacitor.

As $E_p \propto E_0$
 $\therefore E_p = \frac{1}{k} E_0$
 $\Rightarrow V = Ed = \frac{Qd}{\epsilon_0 AK} = \frac{V_0}{K}$

Thus, capacitance is

$$C = \frac{Q}{\frac{V_0}{K}} = \frac{KQ}{V_0} = KC_0 \Rightarrow \frac{C}{C_0} = K$$
 (1)

K is a factor (>1) by which the capacitance is increased. (1)